Durable good monopoly with network effects

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Introduction to harvesting models

Standard approach: The natural resource is located at a single point in space.

- Behringer & Upmann (2014): Harvesting of a *spatial renewable resource*.

- Harvesting requires movement in space.
- Two controls: volume of harvesting and speed of movement.

Questions:

- What does the *optimal path of harvesting and movement* look like?
- Which policy measures can be undertaken to conserve the resource, i.e. to ensure a *sustainable usage* of the resource.

The Programme

Dynamic optimization: simultaneous choice of optimal speed $\{v(t)\}_{t\in\mathcal{T}}$ and harvesting rate $\{h(t)\}_{t\in\mathcal{T}}$.

$$\max_{\{v,h\}} \int_0^T e^{-\rho t}\left(h(t) - C(v(t), h(t))\right) dt$$

$s.t$

$$\begin{aligned}
\dot{s}(t) &= v(t), && \forall t \in \mathcal{T} \\
f_t(t,x) &= g(f(t,x)), && \forall t \in \mathcal{T} \setminus J(x), x \in \mathcal{S} \\
f(t^-,x) - f(t^+,x) &= h(t), && \forall t \in J(x), x \in \mathcal{S} \\
h(t) &\in H(t), && \forall t \in \mathcal{T} \\
f(0,x) &= f_{0x}, && \forall x \in \mathcal{S} \\
s(0) &= 0.
\end{aligned}$$

Growth of the resource

- Harvesting reduces only the stock at location $x = s(t)$

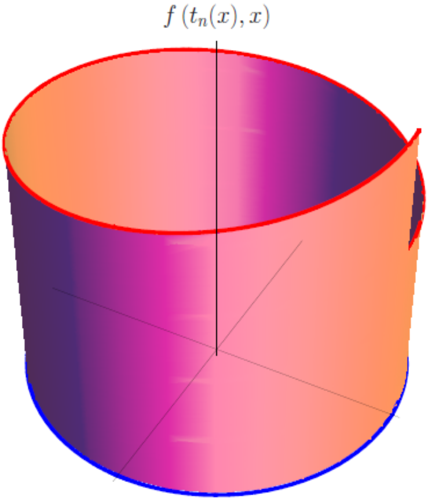
$\rightarrow$ *downward jump* in the stock of the resource.

- $J(x) \equiv \{t_1(x), t_2(x), ...\}$: set of jump times in the stock $f(\cdot, x)$.
- Law of motion for the stock

$$
\begin{aligned}
f_t(t,x) &= g(f(t,x)) && \forall t \in \mathcal{T} \setminus J(x), x \in \mathcal{S}, \\
f(t^-,x) - f(t^+,x) &= h(t) && \forall t \in J(x), x \in \mathcal{S},
\end{aligned}
$$

with $f(0,x) = f_{0x}$ for all $x \in \mathcal{S}$.

Size of the stock of the resource



- Figure 2: Size of the stock of the resource upon arrival: $f(t_n(x), x)$.

Solving the Programme for some fixed location

- Allow for variable harvesting rates $(1-\alpha(t)) \in [0,1]$.
- Consider the harvesting revenue collected *at some fixed location* $x \in \mathcal{S}$ over the planning horizon $\mathcal{T}$. As x is fixed we suppress it.
- t_n : arrival time at location x in the n-th harvesting period,
- $1-\alpha_n \equiv 1-\alpha(t_n)$: the respective harvesting rate.

The present value from the n-th arrival at location x equals

$$y_0(1-\alpha_n)e^{t_n(r-\rho)}\prod_{i=1}^{n-1}\alpha_i,$$

and summing over N periods yields the *aggregate discounted harvesting revenue at x*

$$G(\boldsymbol{\alpha}) = \sum_{n=1}^{n} y_0(1-\alpha_n)e^{t_n(r-\rho)}\prod_{i=1}^{n-1}\alpha_i \tag{1}$$

- Result: $G(\cdot)$ is a multilinear mapping. The optimal value of α_n is either 0 or 1, or it is indeterminate.

Variable Harvesting Rates

The optimal conservation profile is then given by

$$\begin{cases} \boldsymbol{\alpha}^{+} = (1, ..., 1, 0), \text{ if } r - \rho > 0, \\ \boldsymbol{\alpha}^{-} = (0, \bullet, ..., \bullet), \text{ if } r - \rho < 0. \end{cases}$$

A bullet • at a position i implies that α_i is indeterminate and can thus take any value in [0,1].

The corresponding maximized discounted harvest then equals

$$G(\boldsymbol{\alpha}^{+}) = y_0 e^{t_N(r-\rho)} \text{ and } G(\boldsymbol{\alpha}^{-}) = y_0 e^{t_1(r-\rho)}$$

- Result: If $r - \rho > 0$ the agent depletes 100% of the resource in the last round.
  If $r - \rho < 0$ the agent depletes 100% of the resource in the first round.

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The optimal conservation profile is then given by

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- Result: If $r - \rho > 0$ the agent depletes 100% of the resource in the last round.
  If $r - \rho < 0$ the agent depletes 100% of the resource in the first round.
- Remark: If there is only one round of traveling, the first and the last round coincide. $\rightarrow \boldsymbol{\alpha}^+ = (0) = \boldsymbol{\alpha}^-$, there is only one round with 100% depletion irrespective of the sign of $r - \rho$.

Conclusion from Behringer & Upmann (2014)

Model with spatial renewable resource $\rightarrow$ The agent has to move.

- Determination of the optimal harvesting rate, and
- determination of the optimal speed of movement.

Results:
Harvesting takes place in complete rounds.

- The harvesting rate is lower the more rounds are chosen.
- A faster growing resource is harvested at a lower rate.
- The harvesting rate increases with the discount rate.
- Allowing for variable harvesting rates results in total depletion of the resource.

Recent analysis

Model of harvesting a spatial renewable resource can be seen as:

- A monopolist satisfies heterogenous consumers (demand) that generate themselves according to a (stock dependent) *network effect* over time. Satisfied consumers are absent the next time the produce is supplied.

Recent analysis

Model of harvesting a spatial renewable resource can be seen as:

- A monopolist satisfies heterogenous consumers (demand) that generate themselves according to a (stock dependent) *network effect* over time. Satisfied consumers are absent the next time the produce is supplied.
- A monopolist supplies a heterogenous renewable resource to a market. We endogenize prices and look at a non-cooperative game with multiple players.

Aniţa, Behringer, & Upmann (2015) Monopoly

- Note that prices in the above model are fully exogenous.
- In Aniţa, Behringer, and Upmann (2015) we *endogenize prices*.
- The functional $G(\cdot)$ is now:

$$
\begin{aligned}
G(\boldsymbol{\alpha}) \;=\; & \sum_{l=0}^{k-1} \int_0^{\frac{2\pi}{v}} e^{-\rho(t+\frac{2\pi l}{v})} \alpha(t+\frac{2\pi l}{v}) \left(1-\alpha(t+\frac{2\pi l}{v})\right) f_l^{\alpha}(t-)dt \\
& + \int_0^{T-\frac{2\pi k}{v}} e^{-\rho(t+\frac{2\pi k}{v})} \alpha(t+\frac{2\pi k}{v}) \left(1-\alpha(t+\frac{2\pi k}{v})\right) f_k^{\alpha}(t-)dt
\end{aligned}
$$

- This is a monopoly analysis in which at each point in time there is a demand of $1-\alpha$, a *non-durable good.*

Aniţa, Behringer, & Upmann (2015)

- Features of the model are as above:
- Infinite state space, $f(t,x)$ $\forall x \in \mathcal{S}$, but harvesting takes place only a the present location of the agent, $x = s(t)$.
- Choice of speed $\{v(t)\}_{t\in\mathcal{T}}$ determines arrival times at location $x : t_i(x)$ and thus the time of harvesting at location x.
- Jumps in the state variable, $f(t^-,x) - f(t^+,x) = h(t)$, $\forall t \in J(x), x \in \mathcal{S}$. $\rightarrow$ A *differential equation with impulses*.
- Change in the objective to

$$\max_{\{v,h\}} \int_0^T e^{-\rho t} \left(\alpha(t)(1-\alpha(t)) - C(v(t))\right) dt$$

to look at monopoly behaviour.

Aniţa, Behringer, & Upmann (2015)

- The functional is then

$$G(\alpha) =$$
$$\sum_{l=0}^{k-1} \int_0^{\frac{2\pi}{v}} e^{-\rho\left(t+\frac{2\pi l}{v}\right)} \alpha\left(t+\frac{2\pi l}{v}\right)\left(1-\alpha\left(t+\frac{2\pi l}{v}\right)\right) f_l^{\alpha}(t^-)dt +$$
$$\int_0^{t-\frac{2k\pi}{v}} e^{-\rho\left(t+\frac{2\pi l}{v}\right)} \alpha\left(t+\frac{2\pi k}{v}\right)\left(1-\alpha\left(t+\frac{2\pi k}{v}\right)\right) f_k^{\alpha}(t^-)dt$$

- If α^* is an *optimal* control and for any Lebesgue function space $w \in L^\infty(0,T) : 0 \leq \alpha^*(t) + \varepsilon w(t) \leq 1$ $a.e.$ for sufficiently small $\varepsilon > 0$, we have a system in variations as

$$G(\alpha^*) \geq G(\alpha^* + \varepsilon w).$$

Aniţa, Behringer, & Upmann (2015)

- By the recursive structure we have

$$f_l^{\alpha}(t^{+}) = (1-\alpha(t))f^{\alpha}\left(t^{-}\right)$$

where t^{-} denotes the time just before where we take $\alpha(t)$ from the resource. Then

$$f^{\alpha}\left(\left(t+\frac{2\pi}{v}\right)^{+}\right) = e^{r\frac{2\pi}{v}}(1-\alpha(t))f^{\alpha}(t^{-})$$

and

$$f_l^{\alpha}(t^{-}) = f^{\alpha}\left(\left(t+\frac{2\pi l}{v}\right)^{+}\right)$$

and thus for $l \in \{0, 1, ...k\}$

$$\begin{cases} f_{l+1}^{\alpha}(t^{-}) = e^{r\frac{2\pi}{v}}\left(1-\alpha\left(t+\frac{2\pi l}{v}\right)\right)f_l^{\alpha}(t^{-}) \\ f_0^{\alpha}(t^{-}) = e^{rt}f_0(tv). \end{cases}$$

- Let $\lim_{\varepsilon \to 0} \frac{f^{\alpha^*+\varepsilon w} - f^{\alpha^*}}{\varepsilon} \equiv z_l(\cdot)$, then from above we now derive a system *without the impulse problem*

$$0 \geq$$

$$\sum_{l=0}^{k} \int_0^{\frac{2\pi}{v}} e^{-\rho\left(t+\frac{2\pi l}{v}\right)} \left[\begin{array}{c} w\left(t+\frac{2\pi l}{v}\right)\left(1-2\alpha^*\left(t+\frac{2\pi l}{v}\right) f_l^{\alpha^*}(t^-)\right) \\ +\alpha^*\left(t+\frac{2\pi l}{v}\right)\left(1-\alpha^*\left(t+\frac{2\pi l}{v}\right)\right) z_l(t) \end{array} \right] dt$$

where

$$\begin{cases} z_{l+1}(t) = e^{r\frac{2\pi}{v}} \left[\begin{array}{c} -w\left(t+\frac{2\pi l}{v}\right) f_l^{\alpha^*}(t^-) \\ +\left(1-\alpha^*\left(t+\frac{2\pi l}{v}\right)\right) z_l(t) \end{array} \right], t \in \left[0, \frac{2\pi}{v}\right) \\ z_0(t) = 0 \end{cases}$$

- This can be treated as a *dual system* of adjoints of the form,

$$
\begin{aligned}
p_l(t) \quad = \quad & e^{r\frac{2\pi}{v}}\left(1-\alpha^*\left(t+\frac{2\pi l}{v}\right)\right)p_{l+1}(t)+ \\
& e^{-\rho\left(t+\frac{2\pi l}{v}\right)}\alpha^*\left(t+\frac{2\pi l}{v}\right)\left(1-\alpha^*\left(t+\frac{2\pi l}{v}\right)\right)
\end{aligned}
$$

$$
p_k(t)=\begin{cases} e^{-\rho\left(t+\frac{2\pi k}{v}\right)}\alpha^*\left(t+\frac{2\pi k}{v}\right)\left(1-\alpha^*\left(t+\frac{2\pi k}{v}\right)\right),\, t\in\left[\frac{2\pi k}{v},T\right) \\ 0,\, t\in\left[T,\frac{2\pi(k+1)}{v}\right] \end{cases}
$$

Aniţa, Behringer, & Upmann (2015)

- The following conclusions can be derived:

$$
\begin{aligned}
\text{If } \frac{1}{2}\left(1 - e^{\rho\left(t+\frac{2\pi l}{v}\right)+r\frac{2\pi}{v}} p_{l+1}(t)\right) &\in [0,1], \\
\text{then } \alpha^*\left(t + \frac{2\pi l}{v}\right) &= \frac{1}{2}\left(1 - e^{\rho\left(t+\frac{2\pi l}{v}\right)+r\frac{2\pi}{v}} p_{l+1}(t)\right) \\
\text{If } \frac{1}{2}\left(1 - e^{\rho\left(t+\frac{2\pi l}{v}\right)+r\frac{2\pi}{v}} p_{l+1}(t)\right) &< 0, \\
\text{then } \alpha^*\left(t + \frac{2\pi l}{v}\right) &= 0 \\
\text{If } \frac{1}{2}\left(1 - e^{\rho\left(t+\frac{2\pi l}{v}\right)+r\frac{2\pi}{v}} p_{l+1}(t)\right) &> 1 \\
\text{then } \alpha^*\left(t + \frac{2\pi l}{v}\right) &= 1
\end{aligned}
$$

Anița, Behringer, Mosneagu, & Upmann (2015)

- The dual system can be employed in a numerical Matlab analysis.

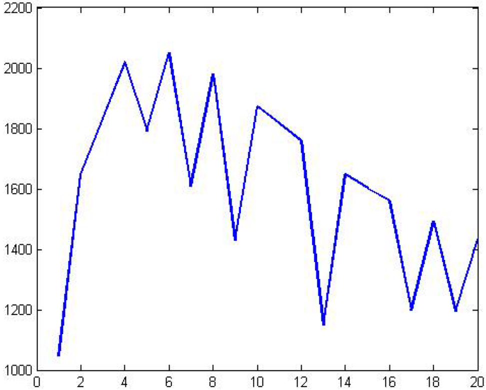
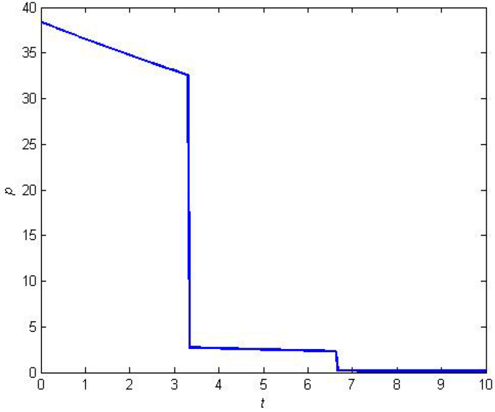
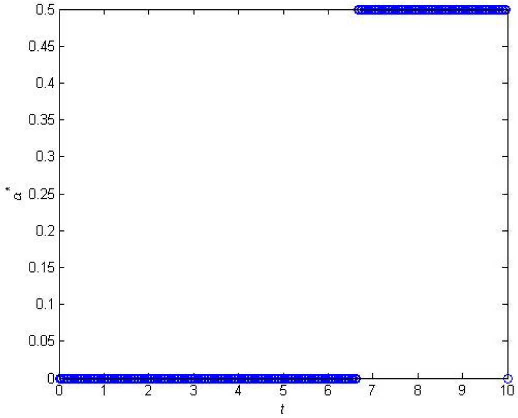


Figure 1: $G(\boldsymbol{\alpha}^*)$ for $T = 10$,
$v = q\pi/10, q = 1, ...20, \rho = 1/20, r = 1, c = 1/1000,\ f_0(x) = |\sin(x)|$
with max obtained at $v = 6\pi/10$.

- At $v = 6\pi/10$ the adjoint $p(t)$ is



- And the optimal $\alpha^*(t)$ is



The "bang-bang" property of the Behringer & Upmann (2014) model prevails for monopoly with non-durable good.

- We now endogenize prices using a linear inverse demand function as for a durable good. What is put on the market in previous rounds (to each consumer or of each variant) decreases price in this round.

Then the cost functional becomes: $G(\boldsymbol{\alpha}) =$

$$\sum_{l=0}^{k-1} \int_0^{\frac{2\pi}{v}} e^{-\rho(t+\frac{2\pi l}{v})} \alpha(t+\frac{2\pi l}{v}) \left(1 - \sum_{j=0}^{l} \alpha(t+\frac{2\pi j}{v})\right) f_l^{\alpha}(t-)dt$$

$$+ \int_0^{T-\frac{2\pi k}{v}} e^{-\rho(t+\frac{2\pi k}{v})} \alpha(t+\frac{2\pi k}{v}) \left(1 - \sum_{h=0}^{k} \alpha(t+\frac{2\pi h}{v})\right) f_k^{\alpha}(t-)dt$$

Solving the Programme for some fixed location

- Putting α_i at some *fixed location* on the market this is sold at a price given by the inverse demand form

$$P(\boldsymbol{\alpha}) = (1 - \sum_{i=1}^{n} \alpha_i)$$

- The discounted revenue from the n-th arrival at location x is then

$$y_0 \alpha_n (1 - \sum_{i=1}^{n} \alpha_i) e^{t_n(r-\rho)} \prod_{i=1}^{n-1} (1 - \alpha_i)$$

and when we sum over n periods we have:

Solving the Programme for some fixed location

- The total revenue functional

$$G(\boldsymbol{\alpha}) = \sum_{n=1}^{n} \left(y_0 \alpha_n (1 - \sum_{i=1}^{n} \alpha_i) e^{t_n (r-\rho)} \prod_{i=1}^{n-1} (1 - \alpha_i) \right) =$$

$$\begin{aligned}
& y_0 \alpha_1 (1 - \alpha_1) e^{t_1 (r-\rho)} + \\
& y_0 \alpha_2 (1 - (\alpha_1 + \alpha_2)) e^{t_2 (r-\rho)} (1 - \alpha_1) + \\
& y_0 \alpha_3 (1 - (\alpha_1 + \alpha_2 + \alpha_3)) e^{t_3 (r-\rho)} (1 - \alpha_1)(1 - \alpha_2) + \\
& \ldots + \\
& y_0 \alpha_n (1 - \sum_{i=1}^{n} \alpha_i) e^{t_n (r-\rho)} \prod_{i=1}^{n-1} (1 - \alpha_i)
\end{aligned}$$

For which we can determine the foncs, with interdependent α_i.

Examples

Example

For $n = 2$ we then have

$$\begin{aligned} G(\boldsymbol{\alpha}) &= y_0\alpha_1(1-\alpha_1)e^{t_1} + y_0\alpha_2(1-(\alpha_1+\alpha_2))e^{t_2}(1-\alpha_1) = \\ & y_0\left(\alpha_1(1-\alpha_1)e^{t_1} + \alpha_2(1-(\alpha_1+\alpha_2))e^{t_2}(1-\alpha_1)\right) \end{aligned}$$

with foncs (set $y_0 = 1$):

$$\frac{\partial G(\boldsymbol{\alpha})}{\partial \alpha_1} = e^{t_1} - 2\alpha_1 e^{t_1} - 2\alpha_2 e^{t_2} + \alpha_2^2 e^{t_2} + 2\alpha_1\alpha_2 e^{t_2} = 0$$

we get

$$\alpha_1 = \frac{e^{t_1} - 2\alpha_2 e^{t_2} + \alpha_2^2 e^{t_2}}{2e^{t_1} - 2\alpha_2 e^{t_2}}$$

Examples

Example

Similarly from

$$\frac{\partial G(\boldsymbol{\alpha})}{\partial \alpha_2} = e^{t_2}\left(\alpha_1 - 1\right)\left(\alpha_1 + 2\alpha_2 - 1\right) = 0$$

we get

$$\alpha_2 = \frac{1}{2} - \frac{1}{2}\alpha_1$$

Solving simultaneously we find

$$\begin{aligned}
\alpha_1^* &= \frac{1}{3e^{t_2}}\left(-4e^{t_1} + 3e^{t_2} + 2\sqrt{e^{t_1}\left(4e^{t_1} - 3e^{t_2}\right)}\right) \\
\alpha_2^* &= \frac{1}{3e^{t_2}}\left(2e^{t_1} - \sqrt{4e^{2t_1} - 3e^{t_1}e^{t_2}}\right)
\end{aligned}$$

Examples

Example

Note that for $e^{t_1} \longrightarrow e^{t_2} (\longrightarrow 1)$ we find

$$\begin{aligned} \alpha_1^* &\longrightarrow \frac{1}{3} \\ \alpha_2^* &= \frac{1}{2} - \frac{1}{2}\alpha_1 \longrightarrow \frac{1}{3} \\ G(\boldsymbol{\alpha})_{n=2} &= \frac{8}{27} \approx 0.3 \end{aligned}$$

Which α is larger in general? Given $\frac{3}{4}e^{t_2} < e^{t_1} < e^{t2}$ roots are non-negative and we can show that $\alpha_1^* < \alpha_2^*$ so that with positive (net) growth, *shares are increasing*.

Examples

Example

For $n = 3$ we then have

$$\begin{aligned} G(\boldsymbol{\alpha}) &= y_0\alpha_1(1-\alpha_1)e^{t_1} + \\ &\quad y_0\alpha_2(1-(\alpha_1+\alpha_2))e^{t_2}(1-\alpha_1) + \\ &\quad y_0\alpha_3(1-(\alpha_1+\alpha_2+\alpha_3))e^{t_3}(1-\alpha_1)(1-\alpha_2) \\ &= y_0\left(\begin{array}{c} \alpha_1(1-\alpha_1)e^{t_1}+ \\ \alpha_2(1-(\alpha_1+\alpha_2))e^{t_2}(1-\alpha_1)+ \\ \alpha_3(1-(\alpha_1+\alpha_2+\alpha_3))e^{t_3}(1-\alpha_1)(1-\alpha_2) \end{array}\right) \end{aligned}$$

from foncs we again find the optimal α_is.

Examples

Example

Note that for $e^{t_1} \longrightarrow e^{t_2} (\longrightarrow 1)$ we find

$$\begin{aligned} \alpha_1^* &\longrightarrow \frac{1}{4} \\ \alpha_2^* &\longrightarrow \frac{1}{4} \\ G(\boldsymbol{\alpha})_{n=3} &= \frac{81}{256} \approx 0.32 \end{aligned}$$

Hence modulo growth we get *Cournot-type solutions* for n rounds of the form

$$\alpha_n = \frac{1}{n+1}.$$

Durable Good Monopoly

- In *Cournot* with $p = a - b\sum x$ and cost $C = cx$, symmetric supply is $x^* = \frac{a-c}{b(n+1)}$ and the equilibrium price converges to the competitive level with more symmetric firms n as

$$\lim_{n\to\infty} p^* = \lim_{n\to\infty}(a - bn(\frac{a-c}{b(n+1)})) = c$$

  the Cournot-Walras convergence.

- If we *add* x *strategic players* to the model then in *symmetric* (zero-growth) equilibrium we have shares of

$$\alpha_{n,x} = \frac{1}{nx+1}.$$

Durable Good Monopoly

- The symmetric equilibrium limit price (with many firms x and/or many rounds n) is then:

$$\begin{aligned}\lim_{n\to\infty} p^* &= \lim_{\substack{n\to\infty \\ \text{or } x\to\infty}} \left(1 - \sum_{y=1}^{x}\sum_{j=0}^{n} \alpha_{j,y}(t + \frac{2\pi j}{v})\right) \\ &= \lim_{\substack{n\to\infty \\ \text{or } x\to\infty}} \left(1 - xn(\frac{1}{nx+1})\right) = 0 = c\end{aligned}$$

so that we have *Cournot convergence* to the Walrasian price (perfect competition).

Conclusion: Durable Good Monopoly

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- The Cournot structure implies that with a durable, spatial-heterogenous good (or with heterogenous consumers and network effects), the monopolist plays against itself each round and thus against its *time-variant copies*.

Conclusion: Durable Good Monopoly

- With growth and flexible shares, optimal output is increasing over time.
- The Cournot structure implies that with a durable, spatial-heterogenous good (or with heterogenous consumers and network effects), the monopolist plays against itself each round and thus against its *time-variant copies*.
- Instead of a *discontinuous jump* as in the non-durable case or as postulated by the *Coase conjecture* for models in continuos time (with infinite possibilities to adjust prices) we have a $o(1/nx)$ convergence where n is the number of rounds the monopoly chooses and x the total number of firms.

Aggregate revenue with constant output

One round of cycling yields

$$\begin{aligned} E(1) &= \alpha(1-\alpha)y_0 \int_0^{2\pi} e^{(r-\rho)t_1} dx = \alpha y_0 \int_0^{2\pi} e^{(r-\rho)\frac{\theta x}{2\pi}} dx = \\ & \alpha y_0 \frac{2\pi}{(r-\rho)\theta} \left(e^{(r-\rho)\theta} - 1 \right) \end{aligned}$$

so total discounted revenue in the *n*th period is

$$\begin{aligned} E(n) &= \alpha \left(1 - \sum_{i=1}^{n} \alpha_i \right) (1-\alpha)^{n-1} y_0 \int_0^{2\pi} e^{-\rho(t_n(x))} e^{\left((n-1)r\theta + r\frac{\theta x}{2\pi}\right)} dx = \\ & \alpha \left(1 - n\alpha\right) (1-\alpha)^{n-1} \frac{2\pi y_0}{(r-\rho)\theta} \left(e^{(r-\rho)\theta} - 1 \right) e^{(n-1)(r-\rho)\theta} \end{aligned}$$

Aggregate revenue with constant output

Summing over all periods (setting $\rho = 0$) and adding the final round

$$E(n+1, s(T)) = \alpha\,(1-(n+1)\alpha)\,\alpha^n \frac{2\pi y}{r\theta}\left(e^{r\frac{\theta s(T)}{2\pi}} - 1\right) e^{nr\theta}$$

yields total aggregate revenue as

$$G(\theta, \alpha) = \sum_{i=1}^{n} E(i) + E(n+1, s(T))$$

which can be calculated as

Aggregate revenue with constant output

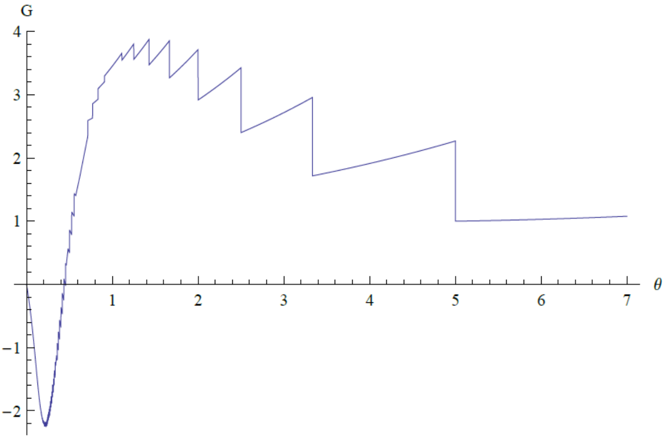
$$G(\theta,\alpha) = \alpha\frac{2\pi y_0}{r\theta} \times$$

$$\left(\begin{array}{c} -\frac{1}{\left(\alpha+e^{-r\theta}-1\right)^2}\times \\ \left(\begin{array}{c} (\alpha-1)\left(2e^{-r\theta}-e^{-2r\theta}-1\right)+ \\ \left(\begin{array}{c} (\alpha-1+Q\alpha)\,e^{r\theta(Q+1)}+ \\ \left(Q\alpha^2-2\alpha+2-2Q\alpha\right)e^{r\theta(Q+2)}+ \\ \left(\alpha-1-Q\alpha^2+Q\alpha\right)e^{r\theta(Q+3)} \end{array} \right)\times \\ e^{3(-r\theta)}\left(-\alpha+1\right)^Q \end{array} \right) \\ +\left(1-(Q+1)\alpha\right)\alpha^Q\left(e^{r\bmod(T,\theta)}-1\right)e^{Qr\theta} \end{array} \right)$$

where $Q = \left\lfloor \frac{T}{\theta} \right\rfloor$.

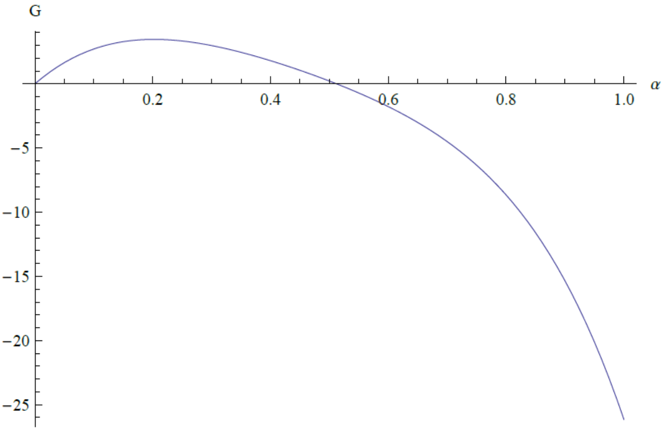
Example

For $T = 10$, $r = r - \rho = 3/20$, $y_0 = 1$, we find the following plot for $G(\theta, \alpha = 1/10)$ in θ



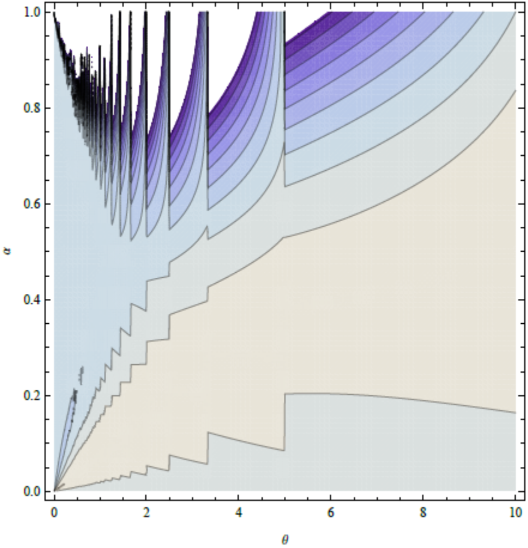
Example

For $T = 10$, $r = r - \rho = 3/20$, $y_0 = 1$, we find the following plot for $G(\theta = 3, \alpha)$ in α



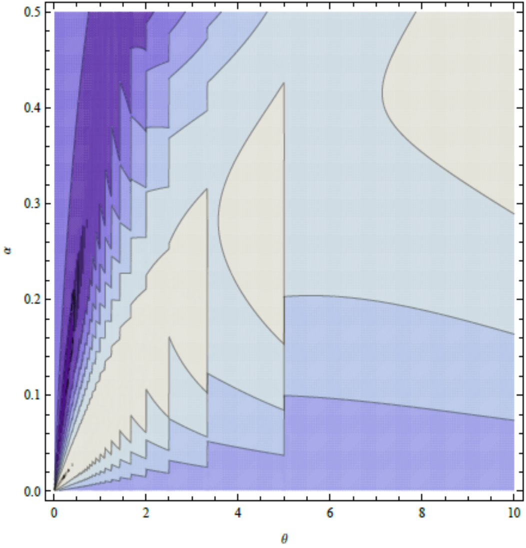
Example

We find the following plot for $G(\theta, \alpha)$ in α, θ



Example

Zooming in $G(\theta, \alpha)$ in α, θ we find



Conclusion Resource Extinction

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- Endogenizing prices thus presents an alternative to forcing the agent to go multiple rounds (or requiring a minmum speed) in a model without prices.
- It remains to be shown how the endogenous prices for durable goods affect the fully dynamic model.

Related Literature

Close to our JEDC paper is: Anton Belyakov, Alexey Davydov, and Vladimir Veliov, (2014): "Optimal cyclic exploitation of renewable resources", 2015, *Journal of Dynamical and Control Systems*. The authors also investigate a circular harvesting model.
The model differs from ours as:

- harvesting intensity is directly determined by the speed of the agent (a "search setting")
- space can be heterogenous
- growth is logistic
- the harvesting agent can remain idle at the beginning of each round
- time goes to infinity
- average revenue is maximized.

Related Literature

- Behringer, Kort, & Upmann (2015)
- The JEDC 2014 model with an uncountable number of state variables (each point of the periphery has its own state variable) proved to be too complex to be solved for any profile of movement. We thus want to limit the number of locations of the resource to a *finite* number.
- This implies that there is some type of a "desert" where no resource is present.
- The agent has to cross the desert before getting to the next location of the resource.
- This non-homogenous distribution of the resource makes movement more interesting, as the time needed for crossing the desert represents a waste.

Related Literature

- While we assumed "en passant" harvesting in the JEDC paper, we dispense with this assumption here and may allow for harvesting to be time-consuming.
- The agent has to reduce his speed, to stop and to stay put for some time period in order to harvest.
- The model allows for exponential and logistic growth, can be decomposed and analytical solutions are obtained.